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The Scalar Fields with Negative Kinetic Energy, Dark Matter and Dark Energy

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The inhomogeneous cosmological model with generalized nonstatic Majumdar-Papapetrou metric is considered. The scalar field with negative kinetic energy and some usual matter sources of the gravitational field such as two-component nonlinear sigma model and perfect fluid are presented. Some exact solutions in these models are obtained and analyzed. In particular it is shown that the latent mass effect and effect of accelerating expansion (quintessence) of the Universe exist in these models. The 5-dimensional generalization of the model are presented, too.

KEY WORDS: Universe; Majumdar-Papapetrou metric; dark matter.

1. INTRODUCTION

One of the more important events in the recent years in observable cosmology was the discovery of an accelerating expansion of the Universe in the modern epoch. Now this fact is connected with some special form of a matter which creates repulsion (negative pressure) in the usual matter. This matter now is called a vacuum energy or a dark energy or a quintessence in accordance with a type model of such form of matter [1, 2]. With the cosmological point of view the dark energy (quintessence) is the main element of the Universe as a kind of matter giving the most contribution in its energy density.

The second main element of the Universe is the dark matter. The term "dark matter" was entered with connection the opening in 40-th years of XX century

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a discrepancy between observational evaluations of gravitating mass and mass of luminous matter in galaxies. As rendered the dark matter is more than the luminous matter in ten times approximately. Dark matter and dark energy now define the way of the Universe evolution. However this forms of matter was not identified exactly. In literature some different kind of models for this matter are presented. The main models for the dark energy considered now is a vacuum (energy of vacuum fluctuations) and some special scalar field ("quintessence"). For the dark matter a some models was suggested too. For example, cold dark matter, hot dark matter, exotic matter, neutrinos, axions, and others. But unified point of view with this issue now is absent.

In this paper we represent the unified model for a dark energy and a dark matter as a development of properties of scalar field with negative kinetic energy (ghost field). Such models with scalar fields having negative kinetic energy have been considered in literature [3, 4, 5, 6] with connection to problem of accelerating modern Universe. In this work we use new approach basing on a model of gravitation field represented with Majumdar-Papapetrou metrics [7]. Majumdar-Papapetrou metrics (MP-metrics) as it was shown in [7] represent the gravitation field of the electrostatic fields. In our work we use the MP-metrics in generalization form. That is helped to construct the cosmological models with ghost field and some usual components of matter such as a perfect fluid.

One of the main properties of a MP-metric (see (2)) connect with following its representation:

$$ds^2 = [1 + F_1(x, y, z, t)]dt^2 - R^2(t)[1 + F_2(x, y, z, t)](dx^2 + dy^2 + dz^2). \quad (1)$$

The quantity $R^2(t)$ can be interpreted as the scale factor, and $F_1(x, y, z, t) = e^{F(x,y,z,t)} - 1$ and $F_2(x, y, z, t) = e^{-F(x,y,z,t)}/R^2 - 1$ as a parameters of scalar metric perturbations. If the functions $F_1, F_2 \to 0$ then the metric (1) converted to the Friedman-Robertson-Walker (FRW) metric. Hence we can use the metric (2) for description of inhomogeneous cosmological dynamics of the Universe [8]. The main element of a material sources of gravitational field is the ghost field. Such ghost-field has the following important properties. That is: i) a difference between dynamical equations for usual scalar field and ghost field detects in second order of perturbation theory only; ii) model consists the effect latent mass (dark matter) explicitly; iii) model consists the effect of dark energy explicitly too; iv) exact geodesic equations are equivalent to the Newtonian equations of a trial particles in a Newtonian gravitation field plus a some small hyrotropic force. The part of this facts was obtained in [11]. The list of these properties are allowed to obtain naturally the unified model of a dark energy and dark matter. In this work we present new results for investigation this model. But we should say that some problems exist in ghost fields models with point view of quantum field theory. This problems was discussed in [12] . The main problem connects with very fast decay of ghost field in spacial quantum process. But in this articles we attempt to show that this process can not influence at main conclusions of suggesting models. This attempt bases on a special rigid linear connection of ghost field with metrics of inhomogeneous space-time.

2. BASIC EQUATIONS OF THE MODEL IN FOUR-DIMENSIONAL MAJUMDAR-PAPAPETROU SPACE-TIME

In this work we will consider the space-time with the metric of the following general form:

$$ds^2 = e^{F(x,y,z,t)}dt^2 - e^{-F(x,y,z,t)}(dx^2 + dy^2 + dz^2). \tag{2}$$

Units are used in which the light velocity $c = 1$ and $x^0 = t$. This metric usually are referred to as a Majumdar-Papapetrou metric. We will use the abbreviation **MP-metric** throughout the paper. The non-vanishing components of Einstein tensor in the space-time (2) are:

$$G_{00} = \frac{1}{2}(F_t)^2 e^{-2F} - \frac{1}{4}[(F_x)^2 + (F_y)^2 + (F_z)^2 - (F_t)^2 e^{-2F}] + \Delta F, \tag{3}$$

$$G_{\alpha\alpha} = -\frac{1}{2}(F_\alpha)^2 + \frac{1}{4}[(F_x)^2 + (F_y)^2 + (F_z)^2 - (F_t)^2 e^{-2F}] - [(F_t)^2 - F_{tt}]e^{-2F}, \tag{4}$$

$$G_{0\alpha} = -\frac{1}{2}F_\alpha F_0 + F_{\alpha,t}, \quad G_{\alpha\beta} = -\frac{1}{2}F_\alpha F_\beta, \quad \alpha \neq \beta = 1, 2, 3; \tag{5}$$

Here

$$F_\alpha = \frac{\partial F}{\partial x^\alpha}, \quad F_t = \frac{\partial F}{\partial x^0}, \quad F_{\alpha,t} = \frac{\partial^2 F}{\partial x^0 \partial x^\alpha},$$

and Δ is 3D Laplace operator.

The matter source of the gravitational field will be presented as a combination of ghost scalar field, perfect fluid and anisotropic electromagnetic radiation, which after average procedure can be considered as additional component of perfect fluid.

The energy-momentum tensor (EM-tensor) will be presented as the sum of the two component:

$$T_{ij} = T_{ij}^{(p)} + T_{ij}^{(0)}, \quad i, j = 0, 1, 2, 3, \tag{6}$$

where

$$T_{ij}^{(p)} = -\phi_i \phi_j + \frac{1}{2} g_{ij} g^{kl} \phi_k \phi_l + V(\phi) g_{ik}.$$

The tensor $T_{ij}^{(p)}$ coincides with the EM-tensor of ghost scalar field. Here κ is the Einstein constant. The tensor $T_{ij}^{(0)}$ can be interpreted as the energy-momentum tensor of a perfect fluid with two components in a co-moving reference system:

$$T_j^{(0)i} = T_j^{(1)i} + T_j^{(2)i}.$$

We consider the first of these components as a usual liquid with an arbitrary equation of state $p = \gamma\varepsilon$: $T_j^{(1)i} = \mathrm{diag}\{\varepsilon, -p, -p, -p\}$. The second component can be presented as an anisotropic unpolarized electromagnetic radiation:

$$T_i^{(2)i} = Q_i, \quad e^{2F} T_\alpha^{(2)0} = -T_0^{(2)\alpha} = S_\alpha, \quad T_\beta^{(2)\alpha} = 0, \quad \alpha, \beta = 1, 2, 3.$$

Really, the energy-momentum tensor for electromagnetic radiation can be written in the form:

$$T_j^{(em)i} = -F^{ik} F_{jk} + \frac{1}{2}\delta_j^i F^{lk} F_{lk},$$

where F_{ik} is a Maxwell tensor of the electromagnetic field [13]. Entering designations

$$E_\alpha = F_{0\alpha}, \quad \alpha = 1, 2, 3; \quad H_3 = F_{12}, \quad H_1 = F_{23}, \quad H_2 = F_{31},$$

we obtain the following components of a tensor of electromagnetic radiation: [13]:

$$\begin{aligned} T_0^{(em)0} &= \frac{1}{2}(\mathbf{E}^2 + \mathbf{H}^2), \quad T_\alpha^{(em)0} = -e^{2F} T_{(0}^{(em)\alpha} = [\mathbf{E} \times \mathbf{H}]_\alpha, \\ T_\alpha^{(em)\beta} &= E_\alpha E_\beta + e^{2F} H_\alpha H_\beta, \quad \alpha, \beta = 1, 2, 3; \qquad (7) \\ T_\alpha^{(em)\alpha} &= -\frac{1}{2}\left(E_\alpha^2 - E_\beta^2 - E_\gamma^2 + e^{2F}\left(H_\alpha^2 - H_\beta^2 - H_\gamma^2\right)\right), \quad \alpha \neq \beta \neq \gamma = 1, 2, 3. \end{aligned}$$

Here we used relations

$$\begin{aligned} E^\alpha &= F^{0\alpha} = g^{00} g^{\alpha\alpha} F_{0\alpha} = E_\alpha, \\ H^\alpha &= F^{\beta\gamma} = g^{\beta\beta} g^{\gamma\gamma} F_{\beta\gamma} = e^{2F} H_\alpha, \quad \alpha \neq \beta \neq \gamma = 1, 2, 3. \end{aligned}$$

In a framework of the cosmological model under consideration it is necessary to consider averaging electromagnetic fields. It is naturally to propose the following properties of averaging components of electromagnetic field [13]:

$$\langle E_\alpha \rangle = 0, \quad \langle H_\alpha \rangle = 0, \quad \alpha = 1, 2, 3, \tag{8}$$

$$\langle E_\alpha E_\beta \rangle = 0, \quad \langle H_\alpha H_\beta \rangle = 0, \quad \alpha \neq \beta = 1, 2, 3, \tag{9}$$

We suggested that statistical fluctuations relate exceptionally to electromagnetic field, but they do not affect to the components of metric. These statements correspond to directed unpolarized electromagnetic radiation. In this case the energy-momentum tensor of electromagnetic radiation $T_i^{(2)j} = \langle T_i^{(em)j} \rangle$ can be written in a following form:

$$T_0^{(2)0} = \rho_1 + \rho_2 + \rho_3, \quad T_\alpha^{(2)\alpha} = -\rho_\alpha + \rho_\beta + \rho_\gamma, \quad \alpha \neq \beta \neq \gamma = 1, 2, 3$$
$$e^{2F} T_0^{(2)\alpha} = -T_\alpha^{(2)0} = S_\alpha, \quad T_\alpha^{(2)\beta} = 0,$$

where $\rho_\alpha(x, y, z, t) = (\langle E_\alpha^2 \rangle + \langle H_\alpha^2 \rangle)/2$ and

$$S_\alpha(x, y, z, t) = \langle [\mathbf{E} \times \mathbf{H}]_\alpha \rangle$$

components of flux of the electromagnetic radiation energy.

For the matter described above and MP-metrics the Einstein equations

$$R_{ik} - \frac{1}{2} g_{ik} R = \kappa T_{ik}$$

can be written in the following form:

$$\phi = -F/\sqrt{2\kappa}, \tag{10}$$

$$\rho_1 = \rho_2 = \rho_3 = \rho, \tag{11}$$

$$\varepsilon + 3\rho = -V(\phi) + \frac{1}{\kappa}\left\{ e^F \Delta F + e^{-F}\left(\frac{\partial F}{\partial t}\right)^2 \right\}, \tag{12}$$

$$p + \rho = -V(\phi) + \frac{e^{-F}}{\kappa}\left\{ -\frac{\partial^2 F}{\partial t^2} + \left(\frac{\partial F}{\partial t}\right)^2 \right\}, \tag{13}$$

$$S_\alpha = F_{,\alpha,t}, \quad \alpha = 1, 2, 3. \tag{14}$$

Here $k = 8\pi G$ is the Einstein gravitational constant and G is the Newtonian gravitational constant.

It is necessary to take into account the field equation which in this case reduced to

$$\frac{1}{2}\frac{\partial^2}{\partial t^2} e^{-2F} - \Delta F = -\sqrt{2\kappa} e^{-F} \frac{\partial V(\phi)}{\partial \phi} \tag{15}$$

The perfect fluid obey the equation

$$\nabla_i T_k^{(1)i} = 0, \quad i, k = 0, 1, 2, 3,$$

which can be reduced to the following form

$$\frac{\partial \varepsilon}{\partial t} = \frac{3}{2}(p + \varepsilon)\frac{\partial F}{\partial t}, \tag{16}$$

$$\frac{\partial p}{\partial x^\alpha} = -\frac{1}{2}(p + \varepsilon)\frac{\partial F}{\partial x^\alpha}, \quad \alpha = 1, 2, 3. \tag{17}$$

The similar equations for $T_k^{(2)i}$ follows from Einstein equations.

The condition of reticence for (17) is the relation

$$p = P(\varepsilon, t).$$

The simplest probability is the equation of state $p = P(\varepsilon)$. In this case, substituting the last relation in (16) and (17) we obtain the following probable functional form of $P(\varepsilon)$:

i) $P(\varepsilon) = -\varepsilon = p = const$;

ii) $P(\varepsilon) = -1/3\varepsilon = -1/3F(x, y, z, t)$;

The spacial probability arises for the following condition iii) $F = A(x, y, z) - a(t)$.

In the case i) perfect fluid is a vacuum, in the case ii) perfect fluid is a inhomogeneous quasi-vacuum and the case iii) corresponds to special cosmological models representing in [11] and, briefly, in the next sections of this work. Now we will derive the equation for $V(\phi)$. Combining the (12), (13) and (15) we obtain

$$\ddot{F} = \frac{1}{2}(\varepsilon - 3p)e^F - e^F\left[V(\phi) + \sqrt{\frac{1}{2\kappa}}\frac{\partial V(\phi)}{\partial \phi}\right]. \tag{18}$$

This equation with field equation (15) display the restriction condition for $V(\phi)$.

We draw the attention of the reader to the following interesting fact. The ghost-field in this model can generate the electromagnetic energy flux S_α where the ghost-field is varied in time and space: $S_\alpha = F_{,\alpha,t}$ without the presence of usual matter. This fact can be important with the observational point of view as a test for such model.

In conclusion of this section we want specially to discuss the problem of fast decay of the ghost fields in quantum process. In this model with agreement to the one of the Einstein equation (10) the ghost field ϕ has rigid connections with metric function F. Let us suggest that a ghost field $\phi \to 0$ as a consequence of quantum decay. In this case the metric must go to a metric of a flat space-time as a consequence of the relation (10). But it is probable if in the space-time any forms of matter is absent except the ghost-field only. We can suggest that the fluctuations of ghost-field decay only and the background ghost-field does not tend to zero. In this case we have to observe the process of a global evolution of a metric to the flat space-time with conservation of a local inhomogeneity of matter and metric. But this process we observe as an accelerating expansion of the Universe. In such a way we have to admit that models with ghost-field is a very useful with this point of view because it corresponds to model of the quintessence. We have to mention that the conclusions about the fast disappearing of ghost fields in real space-time concern the background flat space-time only. In a flat space-time the connection (10) is absent and the decay of ghost field does not influence on a global behavior of the Universe.

3. GEODESIC LINES AND THE GRAVITATION POTENTIAL

The geodesic lines for MP-metric can be represented by equations:

$$\frac{du^\alpha}{ds} = -\frac{1}{2}e^F \frac{\partial F}{\partial x^\alpha} + [\mathbf{u} \times [\mathbf{u} \times \nabla F]]^\alpha, \quad \alpha = 1, 2, 3, \tag{19}$$

$$\frac{du^0}{ds} = -\frac{1}{2}e^{-F} \frac{\partial F}{\partial x^0} + u^0(\mathbf{u}, \nabla F), \tag{20}$$

where we used the normalizing condition for 4-velocity of the test particles:

$$e^F(u^0)^2 - e^{-F}((u^1)^2 + (u^2)^2 + (u^3)^2) = 1.$$

The equation (19) is the usual Newtonian equation of motion of a point particle in the gravitation field with Newtonian potential

$$U = \frac{1}{2}e^F \tag{21}$$

and with additional hyrotropic force $\mathcal{F} = [\mathbf{u} \times [\mathbf{u} \times \nabla F]] = [\mathbf{u} \times [\mathbf{u} \times \nabla \ln(2U)]]$. The force $\mathcal{F}$ does not make the mechanic work. The function $\mathcal{F}$ has the order $(v/c)^2$ (here v is a velocity of particle, c is a speed of light) and for the non-relativistic velocities of particles it is a very small. Therefore the force $\mathcal{F}$ has a small value too. Hence the equation of motion of the test particles obeys for an non-relativistic velocities the Newtonian form:

$$\frac{du^\alpha}{ds} = -\frac{\partial U}{\partial x^\alpha}.$$

The function U is the Newtonian gravitational potential, therefore it must satisfy to Poisson equation

$$\Delta U = 4\pi G\varrho, \tag{22}$$

where ϱ is the mass density of all kinds of matter and G is the Newtonian gravitation constant. Substituting (21) in the left side of (22) we immediately obtain

$$\Delta U = \frac{1}{2}[e^F \Delta F + e^F(\nabla F)^2].$$

By replacing the ΔF in the agreement with (12) we obtain the following relation:

$$\varrho = \varepsilon + 3\rho + \epsilon_\phi.$$

Here

$$\epsilon_\phi = -g^{ik}\phi_i\phi_k + V(\phi) = e^F \sum_{\alpha=1}^{3}(\phi_{,\alpha}) - e^{-F}(\phi_t)^2 + V(\phi),$$

is the total energy of the field ϕ with a negative sign. Hence the Poisson equation takes the following final form:

$$\Delta U = 4\pi G(\varepsilon + 3\rho + \epsilon_\phi). \tag{23}$$

We can see that besides the usual matter with density of energy $\varepsilon + 3\rho$ the energy density ϵ_ϕ of a scalar field ϕ creates a gravitational field. This effect can be interpreted as latent mass effect or a dark matter effect since the field ϕ does not interact directly with the test particles. Hence we can suppose that $\varepsilon + 3\rho$ is a density of luminous component of matter, while ϵ_ϕ is appeared as a dark matter energy density.

4. THE SIMPLE INHOMOGENEOUS COSMOLOGICAL MODEL WITH GHOST-FIELD

Let us choose a special presentation for the function $F(x, y, z, t)$

$$F(x, y, z, t) = A(x, y, z) - a(t). \tag{24}$$

We will use this presentation in this and next sections. This kind of MP-metric simplification and corresponding investigation of the model have been studied in [11]. For this case the MP-metric look as:

$$ds^2 = e^{A+b(t)}dt^2 - e^{-A+a(t)}(dx^2 + dy^2 + dz^2). \tag{25}$$

The function $b(t)$ are introduced for a convenience of a representation and it can be eliminated by a transformation of time $dt \to e^{b(t)/2}dt$. For example without a lost of generality it can be supposed that $b(t) = 0$. In this case we can interpreted the (25) as the metric of inhomogeneous cosmological model with a scale factor

$$R(t) = e^{-a(t)}. \tag{26}$$

Applying the relation (24) to equations of general model we obtain the simplified equations:

$$\Delta A = \sqrt{2\kappa}e^{-A+a}\frac{\partial V(\varphi)}{\partial\varphi} + \left(\frac{\dot b - 3\dot a}{2}\dot a - \ddot a\right)e^{a-b-2A}, \tag{27}$$

$$S_\alpha = F_{,\alpha,t} = 0, \tag{28}$$

$$p = V(\varphi) + g(t)e^{-b-A}, \tag{29}$$

$$\varepsilon = c^2\rho(x, y, z, t) = \frac{1}{\kappa}e^{A-a}\Delta A - V(\varphi) + \frac{\dot a^2}{\kappa}e^{-b-A}, \tag{30}$$

where

$$g(t) = \frac{1}{\kappa}\left[\frac{1}{2}\dot{a}(\dot{b} - \dot{a}) - \ddot{a}\right].$$

Analyzing the equations above, we can make the following conclusions.

The electromagnetic flux of energy S_α in this model equal to zero always. Hence the equations (16) and (17) in this case is the corollary of Einstein equations and we can eliminate their from an explicit representation. Therefore we can suppose $\rho = 0$, then the restrictions on equation of state for usual matter will be absent. Hence we can represent this equation of state in the form: $p = \gamma\varepsilon$. In this case we obtain:

$$\sqrt{\frac{2}{\kappa}}\frac{\partial V(\phi)}{\partial\phi} - \left(\frac{1}{\gamma} + 1\right)V(\phi) = \frac{1}{k}(kg(t) - \dot{a}^2)\left(\frac{1}{\gamma} - 1\right)e^{-A-b}.$$

The general solution of the equation above is

$$V(\phi) = C(t)\exp\{(\gamma+1)\sqrt{2\kappa}\phi/(2\gamma)\} - \frac{1}{k}(kg(t) - \dot{a}^2)e^{-a-b}\exp\{\sqrt{2\kappa}\phi\}, \tag{31}$$

where $C(t)$—is an arbitrary function on t.

The function A does not depend on t, hence the left and the right sides of the field equation (27) should not depend on t too, if we will take in account (31). This condition gives the equation for cosmological evolution of scale factor R [11], that is

$$\frac{1}{k}\left[\ddot{a} - \frac{1}{2}\dot{a}\dot{b} + \frac{3}{2}\dot{a}^2\right]e^{-b} = \sigma_1 = const, \quad \sigma_2 = C(t)e^{\frac{\gamma+1}{2\gamma}a}.$$

Here σ_1 and σ_2 are arbitrary constants. The equation for the gravitational field will have the following form:

$$\Delta A = \sigma_1 e^{-A} + \sigma_2 e^{-(\gamma+1)A/2\gamma}.$$

5. TWO-COMPONENT SIGMA MODEL WITH A GHOST FIELD

The general difficulties of the theories with ghost-fields are in their interpretation. The ghost-field as a self-maintained element of matter cannot be detected in a local experiments. Basing on this fact, we can guess that the ghost-field is the component of more complex system of fields. For example, let us suggest that this field can be the element of a nonlinear sigma model with pseudo-Euclidian metric of a target space [14]. Nonlinear sigma model arises in many modern theories of elementary particles and has many applications in cosmology [15]. In this section we will investigate one of the simplest sigma model with two components and with a pseudo-Euclidean metric of the target space. The pseudo-Euclidean metric of a target space allows us to interpret the one of the chiral field as a usual scalar field and the second field—as a ghost-field.

The action of two-component nonlinear sigma model can be presented by the following way:

$$S = \int \mathcal{L}_\sigma \sqrt{-g}\, d^4x, \tag{32}$$

where

$$\mathcal{L}_\sigma = \frac{1}{2} h_{AB}\, g^{ik} \phi_i^A \phi_k^B. \tag{33}$$

Let the metric of the target space $h_{AB} = \text{diag}\{1, -1\}$ $A, B, \ldots = 1, 2$. Let us also denote the chiral fields as following: $\phi^1 = \psi$, $\phi^2 = \varphi$.

Now we will search the solution of the system of Einstein equations for sigma model (32), considered in the framework of simplified MP-metric (25). Varying the action (32) in respect to the components of a metric tensor g^{ik}, we can obtain the energy momentum tensor of a nonlinear sigma model [14]

$$T_{ik} = h_{AB}\phi^A_{,i}\phi^B_{,k} - \frac{1}{2} g_{ik} h_{CD} \phi^C_{,j} \phi^D_{,l} g^{jl} + g_{ik} V(\psi, \varphi).$$

Considering nonlinear sigma model, we will suggest that usual matter is absent: $\varepsilon = p = \rho = 0$. First of all we consider, in the space-time (25), a non-diagonal part of the Einstein equations

$$R_{ik} - \frac{1}{2} g_{ik} R = \kappa T_{ik},$$

which will have the following view:

$$\frac{1}{2}\frac{\partial a}{\partial t}\frac{\partial A}{\partial x^\xi} = \kappa \frac{\partial \psi}{\partial t}\frac{\partial \psi}{\partial x^\xi} - \kappa \frac{\partial \phi}{\partial t}\frac{\partial \phi}{\partial x^\xi}, \tag{34}$$

$$\frac{1}{2}\frac{\partial A}{\partial x^\xi}\frac{\partial A}{\partial x^\zeta} = -\kappa \frac{\partial \psi}{\partial x^\xi}\frac{\partial \psi}{\partial x^\zeta} + \kappa \frac{\partial \phi}{\partial x^\xi}\frac{\partial \phi}{\partial x^\zeta}, \tag{35}$$

where $\xi, \zeta = 1..3$, $\kappa = 8\pi G/3$. These equations obey the general solutions

$$\psi(x, y, z, t) = \alpha \frac{A(x, y, z) + u(t)}{\sqrt{2\kappa}}, \tag{36}$$

$$\varphi(x, y, z, t) = \beta \frac{A(x, y, z) + v(t)}{\sqrt{2\kappa}}, \tag{37}$$

where the stationary parameters α and β are connected by relation $\beta^2 = 1 + \alpha^2$. They can be interpreted as the constants of interaction. The function $a(t)$ should satisfy to the requirement

$$a(t) = \alpha^2 u(t) + \beta^2 v(t). \tag{38}$$

We note that the obtained equations are the consequences of the model equations.

In this case two diagonal Einstein equations can be written as

$$-\frac{3\alpha^4-\alpha^2}{4}\dot{u}^2-\frac{3\alpha^4+7\alpha^2+4}{4}\dot{v}^2+\frac{3}{2}(\alpha^4+\alpha^2)\dot{u}\dot{v}$$
$$+\kappa e^A V(\psi,\phi)=\Delta A e^{2A-\alpha^2 u+(1+\alpha^2)v}, \qquad (39)$$

$$\alpha^2\ddot{u}+(1+\alpha^2)\ddot{v}+\frac{3\alpha^4+\alpha^2}{4}\dot{u}^2+\frac{3\alpha^4+5\alpha^2+2}{4}\dot{v}^2$$
$$-\frac{3}{2}(\alpha^4+\alpha^2)\dot{u}\dot{v}-\kappa e^A V(\psi,\phi)=0, \qquad (40)$$

where $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$. From the system of equations (13) and (14) it is possible to obtain the important restriction for functions $A(x,y,z)$, $u(t)$ and $v(t)$. The result of combining this equations can be written in form:

$$\Delta A=\left[\alpha^2\ddot{u}-\beta^2\ddot{v}+\frac{1}{2}\alpha^2\dot{u}^2-\frac{1}{2}\beta^2\dot{v}^2\right]e^{\alpha^2 u-\beta^2 v-2A}. \qquad (41)$$

Because the function A depends only on spacial coordinates and does not depend on time, the right part of the expression (41) should not contain also a dependence on t. Therefore

$$\Delta A=\Omega_0 e^{-2A},$$

where

$$\Omega_0=\left[\alpha^2\ddot{u}-\beta^2\ddot{v}+\frac{1}{2}\alpha^2\dot{u}^2-\frac{1}{2}\beta^2\dot{v}^2\right]e^{\alpha^2 u-\beta^2 v}=\text{const}. \qquad (42)$$

Last two equations (42) and (41) describe the cosmological dynamics of Universe. The simplest exact solutions can be obtained in the case when an arbitrary constant $\Omega_0=0$. For this implementation of the following dependencies it is necessary:

$$\ddot{u}+\frac{\dot{u}^2}{2}=\frac{\Omega_1}{\alpha^2},\quad \ddot{v}+\frac{\dot{v}^2}{2}=\frac{\Omega_1}{\beta^2}, \qquad (43)$$

where Ω_1 - an arbitrary constant or a function on time.

The special version of an exact solution of (43) can be obtained when $\Omega_1=0$. This solution reads

$$u(t)=u_0(t)=2\ln\frac{t-t_1}{2},$$
$$v(t)=v_0(t)=2\ln\frac{t-t_2}{2}.$$

Connection between the functions $u(t)$, $v(t)$, $a(t)$ and the equation (26) leads to the power law solution for a scale factor:

$$R(t) = \left(\frac{t}{2}\right)^{\alpha^2+\beta^2} = \left(\frac{t}{2}\right)^{2\alpha^2+1}. \tag{44}$$

Thus, evolution of the scale factor in the model is completely determined by the constants of interaction, α, β, of a gravitational field with the chiral fields ψ, φ (see the picture (5)).

When Ω_1 is the arbitrary constant, the other variant of the exact solution of the Einstein equations can be obtained. The solution of the equation (43) become

$$u(t) = u_0 + 2\ln\left[2\cosh\left(\sqrt{\frac{\Omega_1}{2\alpha^2}}t + \tau_1\right)\right], \tag{45}$$

$$v(t) = v_0 + 2\ln\left[2\cosh\left(\sqrt{\frac{\Omega_1}{2\beta^2}}t + \tau_2\right)\right], \tag{46}$$

where u_0, v_0, τ_1, τ_2 are the integration constants. Scalar fields (45) and (46) behavior showed on the picture 2 (c.). The evolution of a scaler factor displayed by equation

$$R(t) = 2^{\alpha^2+\beta^2}\cosh^{\alpha^2}\left(\sqrt{\frac{\Omega_1}{2\alpha^2}}t\right)\cosh^{\beta^2}\left(\sqrt{\frac{\Omega_1}{2\beta^2}}t\right) \tag{47}$$

It is depended on the parameter Ω_1 and presented on the picture 2 (a.). The speed of Universe expansion is determined by interaction constant α and β too.

Let us consider the effective matter equation of state for this sigma-model. This equation has the form

$$\varepsilon = \frac{1}{2}g^{ik}\phi^A_{,i}\phi^B_{,k}g^{ik} - V(\phi^1,\phi^2) = \frac{1}{2}(\psi_{,j}\psi^{,j} - \varphi_{,j}\varphi^{,j})e^{-a} + V(\psi,\varphi), \tag{48}$$

$$p = \frac{1}{2}g^{ik}\phi^A_{,i}\phi^B_{,k}g^{ik} + V(\phi^1,\phi^2) = \frac{1}{2}(\psi_{,j}\psi^{,j} - \varphi_{,j}\varphi^{,j})e^{-a} - V(\psi,\varphi). \tag{49}$$

First term in (48),(49) decrease and vanish when time tends to infinity. It means, that the nonlinear sigma model with $t \to \infty$ will be characterized by the pseudo-vacuum equation of state $p = -\varepsilon$, corresponding to accelerating expansion of the Universe [16].

For this model we can obtain the effect latent mass using the relation analogical to (23).

6. MODEL IN FIVE DIMENSIONAL SPACE-TIME

The simplified MP-metrics in $d = 5$ space-time has the following form:

$$ds^2 = e^{-A(x,y,z,t)+a(u)} d\sigma^2 - e^{2A(x,y,z,t)+b(u)} du^2. \tag{50}$$

where

$$d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2$$

The matter in this model are represented with following energy-momentum tensor:

$$T_i^k = -\phi_i \phi_k + \frac{1}{2} g_{ik} g^{jl} \phi_j \phi_l + T_{ik}^{(f)},$$

where ϕ—ghost-field, $\phi_i = \partial\phi/\partial x^i$, $T_i^{(f)k} = 0, \quad i \neq k$, $T_\alpha^{(f)\alpha} = -p$, $\alpha = 1, 2, 3$, $T_0^{(f)0} = \varepsilon$, $T_4^{(f)4} = -r$, p—pressure of perfect fluid in 3-D-space-time, ε - energy density of perfect fluid and r—pressure in five (additional) dimension. In this model we suggest that perfect fluid has the anisotropic pressure. The correspondence Einstein equations can be written in a following form:

$$p = \varepsilon = V(\phi) - \frac{3}{4\kappa} e^{-2A-b} (\dot{a}^2 + 2\ddot{a} - \dot{a}\dot{b}) \tag{51}$$

$$r = V(\phi) - \frac{3}{2\kappa} e^{A-a} \diamond A - e^{-2A-b} \frac{9}{4\kappa} \dot{a}^2 \tag{52}$$

$$\phi = \sqrt{\frac{3}{2\kappa}} (-A + a(u)) \tag{53}$$

where

$$\diamond = \frac{\partial^2}{\partial t^2} - \sum_{\alpha=1}^{3} \frac{\partial^2}{\partial x^{\alpha 2}}$$

is the Euclidian D'ALambert's operator. The additional field equation in this case with account (53) has the following view:

$$\diamond A = -e^{-3A+a-b} \left(\ddot{a} + 2\dot{a}^2 - \frac{1}{2} \dot{a}\dot{b} \right) + \sqrt{\frac{2\kappa}{3}} e^{-A+a} \frac{\partial V}{\partial \phi} \tag{54}$$

The equation (51) shows that in 5-dimension the perfect fluid must have the limit rigged state equation for 3D component of pressure.

Combing the field equation (54) and the first Einstein equation (51) we obtain the equation for potential $V(\phi)$:

$$\frac{\partial V}{\partial \phi} + \sqrt{\frac{2\kappa}{3}} (r - V(\phi)) = \frac{1}{2} \sqrt{\frac{3}{2\kappa}} g(u) e^{-2A-b}, \tag{55}$$

where

$$g(u) = 2\ddot{a} + \dot{a}^2 - \dot{a}\dot{b}.$$

As it's followed from the last equation the component of pressure along the five coordinate u are defined by function $V(\phi)$. With other side we can suggest that r connects with energy density ε with usual relativistic relation $r = \gamma\varepsilon$ where $\gamma = \text{const}$. Using this suggestion we obtain the following equation for $V(\phi)$:

$$\frac{\partial V}{\partial \phi} - (1-\gamma)qV(\phi) = \frac{1+\gamma}{2q} g(u) e^{-2a-b} \exp\{2q\phi\},$$

where $q = \sqrt{2\kappa/3}$. In this equation dependence $V(\phi)$ from u can be considered as parametric. Solving this equation we obtain:

$$V(\phi, u) = C(u)\exp\{(1-\gamma)q\phi\} + \frac{1}{2q^2} g(u) e^{-2a-b} \exp\{2q\phi\}.$$

Here $C(u)$–arbitrary function u. Substituting this relation for $V(\phi, u)$ in (51) the state matter equation take the form:

$$\varepsilon = p = C(u)\exp\{(1-\gamma)q\phi\}, \quad r = \gamma C(u)\exp\{(1-\gamma)q\phi\}.$$

In the case $\gamma = 1$ when pressure is isotropic it not dependence from ϕ and varying along u or is constant.

This solution for $V(\phi)$ allows to write the equation for A in following form:

$$\Diamond A = \frac{1}{2} e^{-3A+a-b}[g(u) - 3\dot{a}^2] + q^2(1-\gamma)C(u)\exp\{(2-\gamma)(-A + a(u))\}. \tag{56}$$

Because the function $A = A(x, y, z, t)$ depend from t, x, y, z only and not depend from u the right side from (56) must not depend from u too. Therefore we obtain the following conditions for integrability of (56):

$$\frac{1}{2} e^{a-b}[g(u) - 3\dot{a}^2] = \sigma_0, \tag{57}$$

$$q^2(1-\gamma)C(u)\exp\{(2-\gamma)a(u)\} = \sigma_1, \tag{58}$$

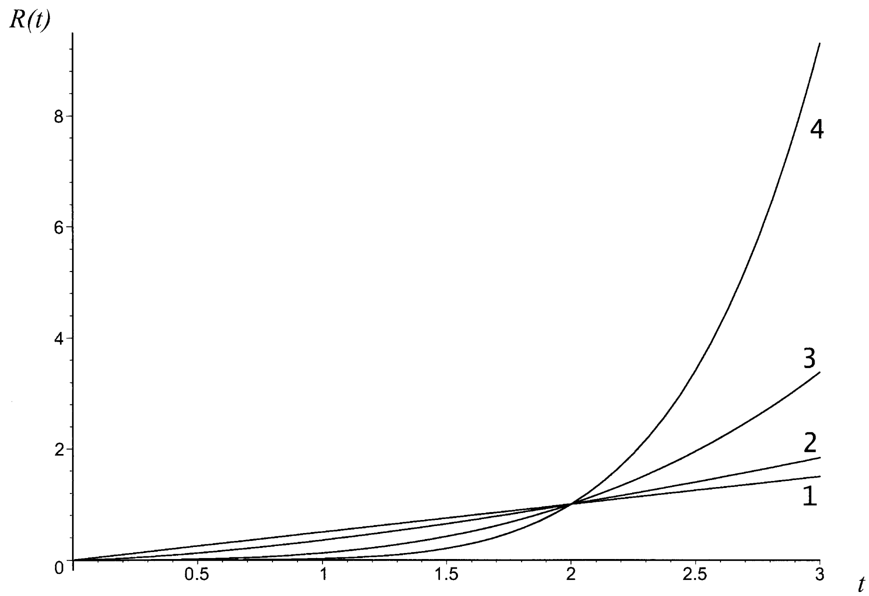
where σ_0 and σ_1–arbitrary real constants. This equation for case $b(u) = 0$ can be written in following view:

$$\ddot{Y} + \sigma_0 Y^2 = 0, \tag{59}$$

$$C(u) = \frac{\sigma_1}{q^2(1-\gamma)} e^{-(2-\gamma)a(u)} \tag{60}$$

Here $Y(u) = e^{-a(u)}$. The correspondence equation for A take the form

$$\Diamond A = \sigma_0 e^{-3A} + \sigma_1 e^{-(2-\gamma)A}. \tag{61}$$



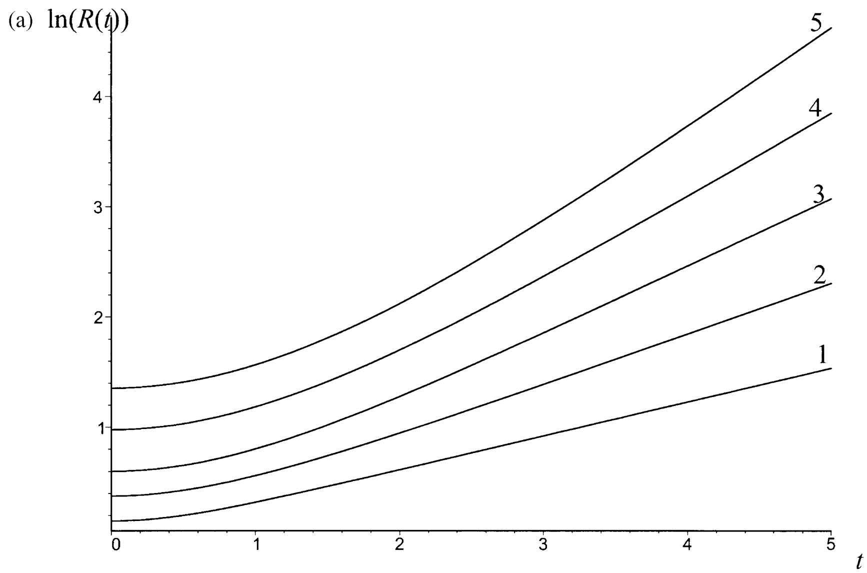
A2

Figure 1. Power law evolution of the scale factor (44) in the case, when $\Omega_1 = 0$. $1 - \alpha = 0$; $2 - \alpha = 0.5$; $3 - \alpha = 1$; $4 - \alpha = 1.5$.

In a particulary when $\sigma_1 = 0$ this equation is the 4D Liouville equation. The exact solutions of Liouville equations can be found with using the method suggested in [17]. When the $\sigma_0 = \sigma_1 = 0$ this is the D'Alembert equation. In a both case dynamic of gravitational field close connected with function A can represent the complex dynamics of a many bodies systems. But in this case it is necessary to define the way indicate the physical hypersurface in the 5-dimension space-time. The way such as Kaluza-Klein theory in this case is not unique but in this papers we not represent this problems.

7. CONCLUSION

In this work it was shown that the model with scalar ghost-field and other components of matter can explain the two main facts from modern cosmology that is the dark matter and accelerating expansion of the Universe. Simultaneously in such models the probability exists to obtain the representation of a inhomogeneous structure of a matter distribution forming in a nonlinear regime. The main problem of the model considered here with simplified MP-metrics consists in a too simplified time dynamics of the gravitational field. As can be seen from the results of the two last sections, the function $A = A(x, y, z)$ does not depend on t.



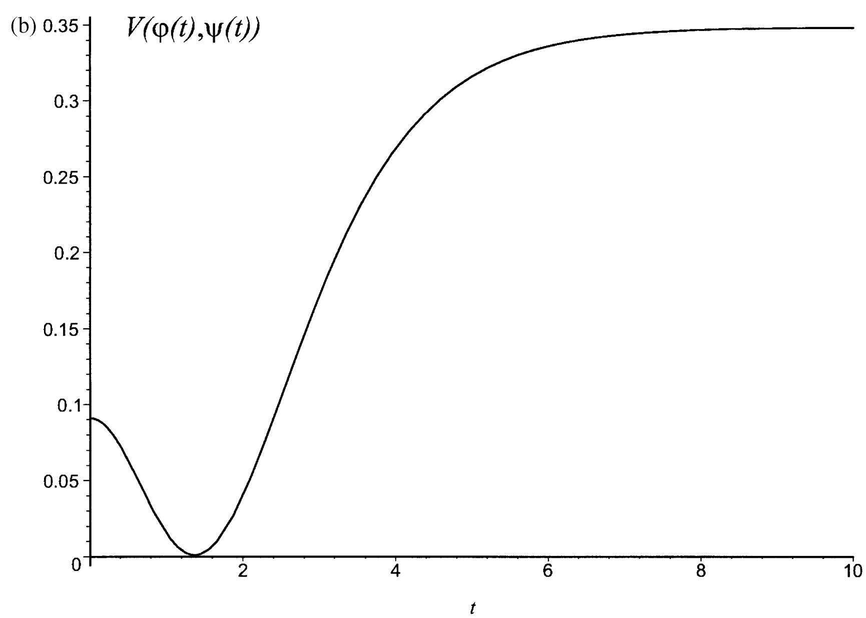


Figure 2. a). –Exponential dependence of the scale factor for $\Omega = 1$. $1 - \alpha = 0.5, \beta = 0.5$; $2 - \alpha = 0.5, \beta = 1$; $3 - \alpha = 1, \beta = 1$; $4 - \alpha = 1, \beta = 1.5$; $5 - \alpha = 1.5, \beta = 1.5$; b). –Time dependence of self-action potential $V(\varphi, \psi)$; c). –Time dependence of scalar fields φ ψ; d).–Time dependence for scalar fields φ and ψ first derivatives on time .

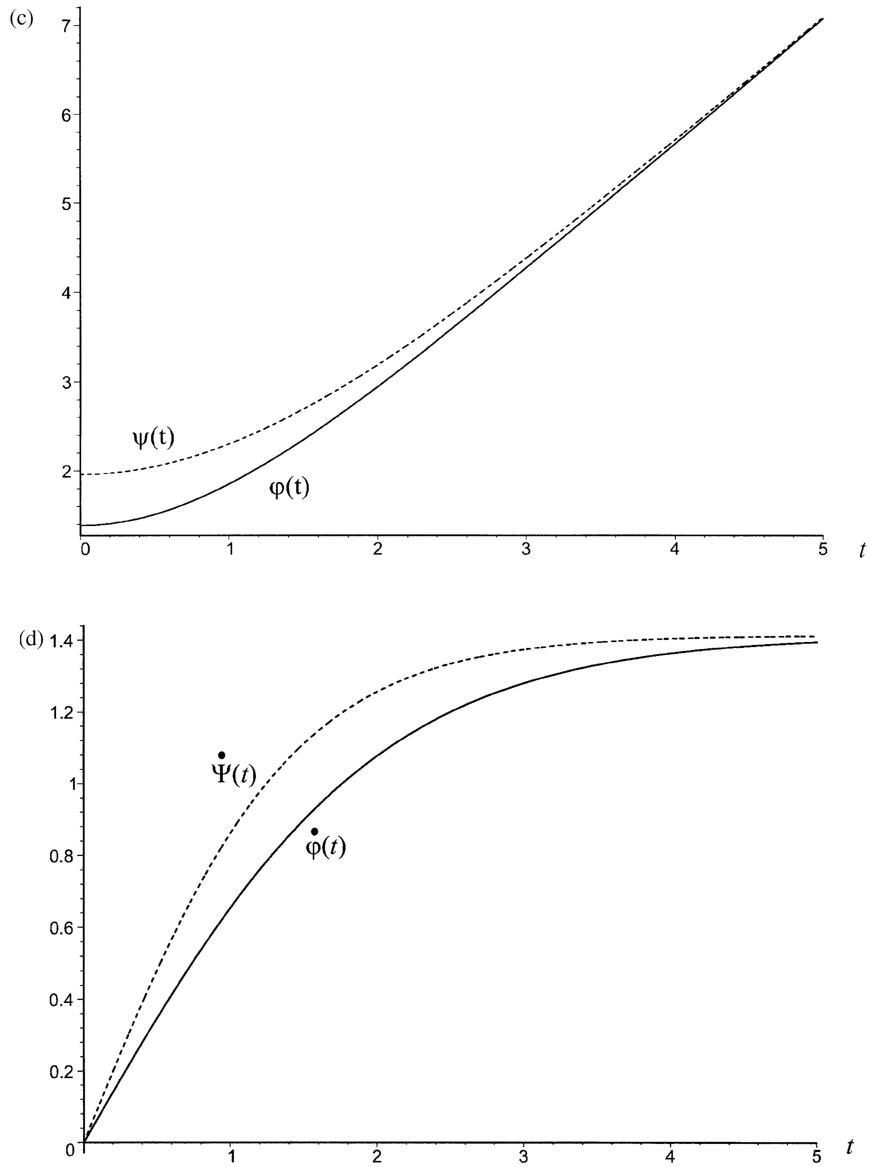


Figure 2. Contined

Hence the dynamics of the gravitational field in such models cannot explain the complex dynamics of separate galaxies, stars and particles. Therefore it is necessary to consider the model without reduction metric (2) or this model should be generalized. In the first case it is difficult to obtain an appropriate dynamics for localized objects. Hence the second way looks more preferable. The one the way to obtain the more general theory is to consider the 5-dimension gravitation theory with MP-metrics and ghost-field. The simplest way to do this was presented in the last section. It can be considered as a very interesting approach to construct the unified cosmological theory with dark matter, dark energy and usual matter.

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